Fault Tree Analysis (Minimal Cutset)

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Basic Set Theory
Sets

- **Definition.** A Set is any well defined collection of “objects.”
- **Definition.** The elements of a set are the objects in a set.
- **Notation.** Usually we denote sets with upper-case letters, elements with lower-case letters. The following notation is used to show set membership

\[ x \in A \] means that \( x \) is a member of the set \( A \)

\[ x \notin A \] means that \( x \) is not a member of the set \( A \).
Ways of Describing Sets

List the elements

\[ A = \{1,2,3,4,5,6\} \]

Verbal description: “A is the set of all integers from 1 to 6, inclusive”

Mathematical inclusion rule:

\[ A = \{ \text{Integers } x | 1 \leq x \leq 6 \} \]

Special Set

Null Set or Empty Set.
This is a set with no elements, often symbolized by \( \emptyset \)

Universal Set.
This is the set of all elements currently under consideration, and is often symbolized by \( \Omega \)
Definition. Subset.
“A is a subset of B” \[ A \subseteq B \]

We say “A is a subset of B” if all the members of A are also members of B.
\[ x \in A \implies x \in B \]

Definition. Proper Subset.
“A is a proper subset of B” if all the members of A are also members of B, but in addition there exists at least one element \( c \) such that \( c \in B \) but \( c \notin A \).
Sets

The **union** of two sets of $X$, $Y$ is the set that contains all elements that are either in $X$ or in $Y$ or in both, is written as $A \cup B$
This is similar to the logical “or” operator.

The **intersection** of two sets $X,Y$ is the set that contains all elements that are common to $X$ and $Y$, is written as $A \cap B$
This is similar to the logical “AND” operator.

The **complement** of a set $X$ is the set that contains all elements that are not in $X$, is written $\bar{A}$
This is similar to the logical “NOT” operator.
Example

\[ \Omega = \{1, 2, 3, 4, 5, 6\} \]

\[ A = \{1, 2, 3\} \quad B = \{3, 4, 5, 6\} \]

\[ A \cap B = \{3\} \quad A \cup B = \{1, 2, 3, 4, 5, 6\} \]

\[ B - A = \{4, 5, 6\} \quad \overline{B} = \{1, 2\} \]
Venn Diagrams

- Venn Diagrams use topological areas to stand for sets. I’ve done this one for you.
## Boolean Algebra

<table>
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<tr>
<th>Rule</th>
<th>Mathematical Form</th>
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<tr>
<td><strong>Idempotent Rule</strong></td>
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<tr>
<td>A.A = A</td>
<td>A ∩ A = A</td>
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<tr>
<td>A + A = A</td>
<td>A ∪ A = A</td>
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<tr>
<td><strong>Absorption Rule</strong></td>
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<td>A.(A + B) = A</td>
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<td><strong>Commutative Rule</strong></td>
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<td>A.B = B.A</td>
<td>A ∩ B = B ∩ A</td>
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<td>A + B = B + A</td>
<td>A ∪ B = B ∪ A</td>
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<tr>
<td><strong>Associative Rule</strong></td>
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<tr>
<td>A.(B.C) = (A.B).C</td>
<td>A ∩ (B ∩ C) = (A ∩ B) ∩ C</td>
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<tr>
<td>A + (B + C) = (A + B) + C</td>
<td>A ∪ (B ∪ C) = (A ∪ B) ∪ C</td>
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<tr>
<td><strong>Distributive Rule</strong></td>
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</tr>
<tr>
<td>A.(B + C) = A.B + A.C</td>
<td>A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)</td>
</tr>
</tbody>
</table>
CUT SET
Cut Set

- A cut set in a fault tree is a set of basic events whose (simultaneous) occurrence ensures that the TOP event occurs.

- A cut set is said to be a minimal cut set if, when any basic event is removed from the set, the remaining events collectively are no longer a cut set.
Minimal Cut Set

- **Minimal cut set** analysis rearranges the fault tree so that any basic event that appears in different parts of the fault tree is not "double counted" in the quantitative evaluation.

- The result of **minimal cut set** analysis is a new fault tree, logically equivalent to the original, consisting of an OR gate beneath the top event, whose inputs are the **minimal cut sets**.

- Each **minimal cut set** is an AND gate containing a set of basic inputs necessary and sufficient to cause the top event.

- So, the resulting cut set is a **Minimal Cut set** which no subset is also a cut set.

- Boolean Algebra is used for the analyses.
Example 1  –  Pumping of Water

- Water is pumped automatically from the supply tank to the process.
- When the regulator is energized, one of the pumps is started and acid passes through the feed pipes; if no water is detected in the feed pipe the second pump is started.
Example 1 – Cut set

A CUT SET is a combination of basic events which will produce TOP EVENT

M, M.Z, W.M, W.Z are all cut set

Why?

Because, from the base element, the path will lead to the top event

Minimal CUT SET is a CUT SET if any basic event is removed the TOP EVENT will not occur
Example 1 – Cut Set

A CUT SET = combination of basic events which will produce TOP EVENT

In this example: M, M.Z, W.M, W.Z are all cut set. But Minimal CUT SET M and W.Z

Idempotent  \( A+A=A \)
\( A.A=A \)

Absorption  \( A+A.B=A \)
\( A.(A+B)=A \)

For example:
\[(M+W) \cdot (M+Z) \]
\[= M.M + M.Z + W.M + W.Z \]
\[= M + M.Z + W.M + W.Z \]
\[= (M + M.Z + M.W) + W.Z \]
\[= M + W.Z \]
Example 1 – Minimal Cut Set

Original Cut set

- PUMP FAIL
  - PUMP A FAILS
    - Failure of Power Supply
      - M
    - Pump A Mechanical Failure
      - W
  - Pump B FAILS
    - Failure of Power Supply
      - M
    - Pump B Mechanical Failure
      - Z

Minimal Cut set

- PUMP FAIL
  - Failure of Power Supply
    - M
  - Mechanical Failure of Pumps
    - Pump A Mechanical Failure
      - W
    - Pump B Mechanical Failure
      - Z

P = (M + W)(M + Z)
P = M + (W . Z)
Example 2 – Power Supply

- A power supply system consists of the following elements
  - An Off site power supply
  - A backup power supply which consists of a diesel generator and an automatic transfer switch
  - The system ‘fails’ upon blackout – when power is not available
  - Blackout occurs when both off-site power and backup power fail

- Draw the fault tree of the system
- Determine the minimum cutset
Example 2 – Power Supply

Blackout

- Offsite Power Supply
  - Basic event 1

- Backup Power Supply
  - Transfer Switch Failure
  - Diesel Generator Failure

Event not fully developed but treated as basic event

CUTSET: \{A,B,C\}, \{A,B\}, \{A,C\}

Minimal Cutset: \{A,B\}, \{A,C\}
Example 3 – Minimal Cut Set

\[(A + B) \cdot [ (C + D) \cdot (E + C) + (D \cdot E) ]\]
Example 3 – Minimal Cut Set

\[(A + B) \cdot [(C + D) \cdot (E + C) + (D.E)]\]
\[= (A + B) \cdot (C.E + D.E + C.C + D.C + D.E)\]
\[= (A + B) \cdot (C + C.E + D.E + D.C + D.E)\]
\[= (A + B) \cdot (C + C.D + C.E + D.E + D.E)\]
\[= (A + B) \cdot (C + C.E + D.E)\]
\[= (A + B) \cdot (C + D.E)\]

IDEMPOTENT LAW & ABSORPTION LAW
are applied
Class Workshop
Show that the two fault tree are equivalent
Class Workshop

Draw the Minimal Cut Set Fault tree for the given fault tree
END OF LECTURE